Type I background fields in terms of type IIB ones

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We choose such boundary conditions for open IIB superstring theory which preserve N=1 SUSY. The explicite solution of the boundary conditions yields effective theory which is symmetric under world-sheet parity transformation $\Omega:\sigma\to -\sigma$. We recognize effective theory as closed type I superstring theory. Its background fields,beside known Ω even fields of the initial IIB theory, contain improvements quadratic in Ω odd ones.

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I. INTRODUCTION

There are five consistent supersymmetric string theories: type I, type IIA and IIB and two heterotic string theories. They are related by web of dualities and in fact provide different descriptions of the same theory [1]. In this article we are going to improve known relation between type I and type IIB theories.

It is known that type I superstring theory can be obtained from type IIB superstring theory as a projection on the states that are even under world-sheet parity transformation $\Omega:\sigma\to-\sigma$. After this orientifold projection the following background fields survive: graviton $G_{\mu\nu}$ and dilaton Φ from NS-NS sector, the sum of two same chirality gravitions $\psi^{\alpha}_{+\mu}=\psi^{\alpha}_{\mu}+\bar{\psi}^{\alpha}_{\mu}$ and dilatinos $\lambda^{\alpha}_{+}=\lambda^{\alpha}+\bar{\lambda}^{\alpha}$ from NS-R sector and two rank antisymmetric tenzor $C_{\mu\nu}$ from R-R sector. The states that are odd under Ω transformations: antisymmetric tensor $B_{\mu\nu}$ from NS-NS sector, difference of two gravitinos $\psi^{\alpha}_{-\mu}=\psi^{\alpha}_{\mu}-\bar{\psi}^{\alpha}_{\mu}$ and dilatinos $\lambda^{\alpha}_{-}=\lambda^{\alpha}-\bar{\lambda}^{\alpha}$ from NS-R sector, and scalar C_0 and four rank antisymmetric tensor $C_{\mu\nu\rho\sigma}$ with self dual field strength from R-R sector, are eliminated by above projection.

We are going to investigate Green-Schwarz formulation of type IIB superstring adopted by Refs.[2]. In particular, we use the form of Ref.[3] which corresponds to constant graviton $G_{\mu\nu}$, antisymmetric field $B_{\mu\nu}$, two gravitinos ψ^{α}_{μ} and $\bar{\psi}^{\alpha}_{\mu}$, and R-R field strength $F^{\alpha\beta}$. In this approach dilaton Φ and two dilatinos λ^{α} and $\bar{\lambda}^{\alpha}$ are set to zero.

Let us express connection between two descriptions of R-R sector [1]. It is known that bispinor $F^{\alpha\beta}=iS^{\alpha}(\Gamma^{0}\tilde{S})^{\beta}$, made from same chirality spinors S^{α} and \tilde{S}^{α} , can be expanded into complete set of antisymmetric gamma matrices

$$F^{\alpha\beta} = \sum_{k=0}^{D} \frac{i^k}{k!} F_{(k)} \Gamma_{(k)}^{\alpha\beta}, \quad \left[\Gamma_{(k)}^{\alpha\beta} = (\Gamma^{[\mu_1 \dots \mu_k]})^{\alpha\beta} \right] \quad (1)$$

where in short-hand notation $F_{(k)}$ is k-rank antisymmet-

ric tensor. As a consequence of chirality conditions on bispinor $F^{\alpha\beta}$ and duality relations, the independent tensors are $F_{(1)}$, $F_{(3)}$ and self-dual part of $F_{(5)}$. Using physical state condition these tensors can be solved in terms of potentials $F_{(k)} = dC_{(k-1)}$, so that IIB theory contains just the potentials $C_{(0)}$, $C_{(2)}$ and $C_{(4)}$. Note that symmetric part of $F^{\alpha\beta}$, $F^{\alpha\beta}_s = \frac{1}{2}(F^{\alpha\beta} + F^{\beta\alpha})$, corresponds to the potentials C_0 and $C_{\mu\nu\rho\sigma}$, and antisymmetric part, $F^{\alpha\beta}_a = \frac{1}{2}(F^{\alpha\beta} - F^{\beta\alpha})$, corresponds to the potential $C_{\mu\nu}$.

We start with type IIB open superstring theory described by bosonic coordinates x^{μ} , and same chirality fermionic ones θ^{α} and $\bar{\theta}^{\alpha}$. According to Refs.[4]-[5] we apply the canonical method, treating boundary conditions as the canonical constraints. We choose Neumann boundary conditions for bosonic coordinates x^{μ} and $(\theta^{\alpha} - \bar{\theta}^{\alpha})|_{0}^{\pi} = 0$ for fermionic coordinates in order to preserve N=1 SUSY from initial N=2 SUSY in type IIB theory. We are able to solve all boundary conditions and obtain effective theory. It occurs that this effective theory, even under world-sheet parity transformation Ω , is just type I closed superstring theory defined on orientifold projection. The main result of the article contains generalized expressions of type I superstring background fields in terms of type IIB ones

$$G_{\mu\nu} \rightarrow G_{\mu\nu}^{eff} = G_{\mu\nu} - 4B_{\mu\rho}G^{\rho\sigma}B_{\sigma\nu} , B_{\mu\nu} \rightarrow 0 ,$$

$$\frac{1}{2}\psi_{+\mu}^{\alpha} \rightarrow (\Psi_{eff})_{\mu}^{\alpha} = \frac{1}{2}\psi_{+\mu}^{\alpha} + B_{\mu\rho}G^{\rho\nu}\psi_{-\nu}^{\alpha} , \psi_{-\mu}^{\alpha} \rightarrow 0 ,$$

$$F_{\mu\nu\rho} \rightarrow F_{\mu\nu\rho}^{eff} = F_{\mu\nu\rho} - \frac{1}{D}G^{\epsilon\eta}\psi_{-\epsilon}\Gamma_{[\mu\nu\rho]}\psi_{-\eta} ,$$

$$F_{\mu} \rightarrow 0 , F_{\mu\nu\rho\sigma\epsilon} \rightarrow 0 . \tag{2}$$

Therefore, fields $B_{\mu\nu}$ and $\psi^{\alpha}_{-\mu}$, which describe states odd under world-sheet parity transformations, are not complitely eliminated, but they survive as quadratic terms in type I superstring background.

We will refer to the fields $G_{\mu\nu}$, $B_{\mu\nu}$, ψ^{α}_{μ} , $\bar{\psi}^{\alpha}_{\mu}$ and $F^{\alpha\beta}$ (or $F_{(1)}$, $F_{(3)}$ and $F_{(5)}$) as type IIB background fields and to fields $G^{eff}_{\mu\nu}$, $(\Psi_{eff})^{\alpha}_{\mu}$ and $F^{\alpha\beta}_{eff}$ (or $F^{eff}_{(3)}$) as type I background fields (the background fields seen by type IIB and type I superstring respectively). The names are taken from initial and final (effective) descriptions of the same theory.

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II. CANONICAL APPROACH TO GREEN-SCHWARZ FORMULATION OF IIB THEORY

We will investigate the type IIB superstring theory using Green-Scwarz formulation of Ref.[3]. The action is

$$S = \kappa \int_{\Sigma} d^{2}\xi \left[\frac{1}{2} \eta^{ab} G_{\mu\nu} + \varepsilon^{ab} B_{\mu\nu} \right] \partial_{a} x^{\mu} \partial_{b} x^{\nu}$$

$$- \int_{\Sigma} d^{2}\xi \pi_{\alpha} (\partial_{\tau} - \partial_{\sigma}) (\theta^{\alpha} + \psi^{\alpha}_{\mu} x^{\mu})$$

$$+ \int_{\Sigma} d^{2}\xi \left[(\partial_{\tau} + \partial_{\sigma}) (\bar{\theta}^{\alpha} + \bar{\psi}^{\alpha}_{\mu} x^{\mu}) \bar{\pi}_{\alpha} + \frac{1}{2\kappa} \pi_{\alpha} F^{\alpha\beta} \bar{\pi}_{\beta} \right] (3)$$

where the world sheet Σ is parameterized by $\xi^a = (\xi^0 = \tau, \xi^1 = \sigma)$, and D = 10 dimensional space-time is parameterized by coordinates x^{μ} ($\mu = 0, 1, 2, ..., D - 1$). The fermionic part of superspace is spanned by same chirality fermionic coordinates θ^{α} and $\bar{\theta}^{\alpha}$, while the variables π_{α} and $\bar{\pi}_{\alpha}$ are their canonically conjugated momenta.

A. Canonical Hamiltonian and boundary conditions as canonical constraints

According to the definition of canonical Hamiltonian, $\mathcal{H}_c = \dot{x}^{\mu}\pi_{\mu} + \dot{\theta}^{\alpha}\pi_{\alpha} + \dot{\bar{\theta}}^{\alpha}\bar{\pi}_{\alpha} - \mathcal{L}$, we have

$$H_c = \int d\sigma \mathcal{H}_c \,, \quad \mathcal{H}_c = T_- - T_+ \,, \quad T_{\pm} = t_{\pm} - \tau_{\pm} \,,$$
 (4)

where

$$t_{\pm} = \mp \frac{1}{4\kappa} G^{\mu\nu} I_{\pm\mu} I_{\pm\nu} ,$$

$$I_{\pm\mu} = \pi_{\mu} + 2\kappa \Pi_{\pm\mu\nu} x^{\prime\nu} + \pi_{\alpha} \psi_{\mu}^{\alpha} - \bar{\psi}_{\mu}^{\alpha} \bar{\pi}_{\alpha} ,$$

$$\tau_{+} = (\theta^{\prime\alpha} + \psi_{\mu}^{\alpha} x^{\prime\mu}) \pi_{\alpha} - \frac{1}{4\kappa} \pi_{\alpha} F^{\alpha\beta} \bar{\pi}_{\beta} ,$$

$$\tau_{-} = (\bar{\theta}^{\prime\alpha} + \bar{\psi}_{\mu}^{\alpha} x^{\prime\mu}) \bar{\pi}_{\alpha} + \frac{1}{4\kappa} \pi_{\alpha} F^{\alpha\beta} \bar{\pi}_{\beta} ,$$
(5)

and π_{μ} denotes momentum canonically conjugated to the coordinate x^{μ} . Using the standard Poisson bracket algebra we find that components T_{\pm} satisfy Virasoro algebra.

Following method of Ref.[5], using canonical approach, we will derive boundary conditions directly in terms of canonical variables. Varying Hamiltonian H_c we obtain

$$\delta H_c = \delta H_c^{(R)} - \left[\gamma_\mu^{(0)} \delta x^\mu + \pi_\alpha \delta \theta^\alpha + \delta \bar{\theta}^\alpha \bar{\pi}_\alpha \right]_0^\pi, \quad (6)$$

where $\delta H_c^{(R)}$ is regular term without τ and σ derivatives of coordinate variations, δx^{μ} , $\delta \theta^{\alpha}$, $\delta \bar{\theta}^{\alpha}$, and variations of their canonically conjugated momenta, $\delta \pi_{\mu}$, $\delta \pi_{\alpha}$ and $\delta \bar{\pi}_{\alpha}$, while

$$\gamma_{\mu}^{(0)} = \Pi_{+\mu}{}^{\nu} I_{-\nu} + \Pi_{-\mu}{}^{\nu} I_{+\nu} + \pi_{\alpha} \psi_{\mu}^{\alpha} + \bar{\psi}_{\mu}^{\alpha} \bar{\pi}_{\alpha} \,. \tag{7}$$

As a time translation generator canonical Hamiltonian must have well defined derivatives in its variables. Consequently, boundary term has to vanish and we obtain

$$\left[\gamma_{\mu}^{(0)}\delta x^{\mu} + \pi_{\alpha}\delta\theta^{\alpha} + \delta\bar{\theta}^{\alpha}\bar{\pi}_{\alpha}\right]\Big|_{0}^{\pi} = 0.$$
 (8)

For bosonic coordinates x^{μ} we choose Neumann boundary conditions implying

$$\gamma_{\mu}^{(0)}\big|_{0}^{\pi} = 0. \tag{9}$$

In order to preserve N=1 SUSY of the initial N=2 SUSY, following [3], for fermionic coordinates we chose

$$(\theta^{\alpha} - \bar{\theta}^{\alpha})\Big|_{0}^{\pi} = 0, \qquad (\pi_{\alpha} - \bar{\pi}_{\alpha})\Big|_{0}^{\pi} = 0, \qquad (10)$$

where the first condition produces the second one. According to Refs.[4, 5], we will treat the expressions (9)-(10) as canonical constraints.

B. Dirac consistency procedure

As well as in Refs.[4]-[6], Dirac canonical consistency procedure for constraints (9) gives an infinite set of constraints which can be rewritten in the compact σ -dependent form

$$\Gamma_{\mu}(\sigma) = \Pi_{+\mu}{}^{\nu}I_{-\nu}(\sigma) + \Pi_{-\mu}{}^{\nu}I_{+\nu}(-\sigma) + \pi_{\alpha}(-\sigma)\psi_{\mu}^{\alpha} + \bar{\psi}_{\mu}^{\alpha}\bar{\pi}_{\alpha}(\sigma).$$
 (11)

Applying the same procedure for fermionic boundary conditions (10), we obtain respectively

$$\Gamma^{\alpha}(\sigma) = \Theta^{\alpha}(\sigma) - \bar{\Theta}^{\alpha}(\sigma), \ \Gamma^{\pi}_{\alpha}(\sigma) = \pi_{\alpha}(-\sigma) - \bar{\pi}_{\alpha}(\sigma)$$
 (12)

where

$$\Theta^{\alpha}(\sigma) = \theta^{\alpha}(-\sigma) - \psi^{\alpha}_{\mu}\tilde{q}^{\mu}(\sigma) - \frac{1}{2\kappa}F^{\alpha\beta}\int_{0}^{\sigma}d\sigma_{1}P_{s}\bar{\pi}_{\beta} + \frac{1}{2\kappa}G^{\mu\nu}\psi^{\alpha}_{\mu}\int_{0}^{\sigma}d\sigma_{1}P_{s}(I_{+\nu} + I_{-\nu}), \qquad (13)$$

$$\bar{\Theta}^{\alpha}(\sigma) = \bar{\theta}^{\alpha}(\sigma) + \bar{\psi}^{\alpha}_{\mu}\tilde{q}^{\mu}(\sigma) + \frac{1}{2\kappa}F^{\beta\alpha}\int_{0}^{\sigma}d\sigma_{1}P_{s}\pi_{\beta} + \frac{1}{2\kappa}G^{\mu\nu}\bar{\psi}^{\alpha}_{\mu}\int_{0}^{\sigma}d\sigma_{1}P_{s}(I_{+\nu} + I_{-\nu}). \tag{14}$$

We introduced new variables, symmetric and antisymmetric under world-sheet parity transformation $\Omega: \sigma \to -\sigma$. For bosonic variables we use standard notation [4]-[5]

$$q^{\mu}(\sigma) = P_s x^{\mu}(\sigma), \quad \tilde{q}^{\mu}(\sigma) = P_a x^{\mu}(\sigma),$$

$$p_{\mu}(\sigma) = P_s \pi_{\mu}(\sigma), \quad \tilde{p}_{\mu}(\sigma) = P_a \pi_{\mu}(\sigma), \quad (15)$$

while for fermionic ones we use the projectors on σ symmetric and antisymmetric parts

$$P_s = \frac{1}{2}(1+\Omega), \quad P_a = \frac{1}{2}(1-\Omega).$$
 (16)

From $\{H_c, \Gamma_A\} = \Gamma_A' \approx 0, \Gamma_A = (\Gamma_\mu, \Gamma^\alpha, \Gamma_\alpha^\pi)$, it follows that there are no more constraints in the theory.

Also, there exists the set of constraints at $\sigma=\pi$, $\bar{\Gamma}_{\mu}(\sigma)$, $\bar{\Gamma}^{\alpha}(\sigma)$ and $\bar{\Gamma}^{\alpha}_{\pi}(\sigma)$. They differ from conditions (11) and (12) only in terms depending on $-\sigma$ and can be obtained just replacing $-\sigma$ with $2\pi-\sigma$. Constraints at $\sigma=\pi$ can be solved by 2π periodicity of all canonical variables as well as in Refs.[4, 5]. So, the effective theory will be closed string theory.

C. Classification of constraints

Computing the algebra of the constraints ${}^*\Gamma_A = (\Gamma_\mu, \Gamma'^\alpha, \Gamma^\pi_\alpha)$

$$\{{}^{\star}\Gamma_A, {}^{\star}\Gamma_B\} = M_{AB}\delta', \tag{17}$$

we obtain the supermatrix

$$M_{AB} = \begin{pmatrix} -\kappa G_{\mu\nu}^{eff} & 2(\Psi_{eff})_{\mu}^{\gamma} & 0\\ -2(\Psi_{eff})_{\nu}^{\alpha} & -\frac{1}{\kappa} F_{eff}^{\alpha\gamma} & -2\delta^{\alpha}_{\delta}\\ 0 & -2\delta_{\beta}^{\gamma} & 0 \end{pmatrix}, \quad (18)$$

with the effective background fields

$$G_{\mu\nu}^{eff} = G_{\mu\nu} - 4B_{\mu\rho}G^{\rho\lambda}B_{\lambda\nu} ,$$

$$(\Psi_{eff})_{\mu}^{\alpha} = \frac{1}{2}\psi_{+\mu}^{\alpha} + B_{\mu\rho}G^{\rho\nu}\psi_{-\nu}^{\alpha} ,$$

$$F_{eff}^{\alpha\beta} = F_{a}^{\alpha\beta} - \psi_{-\mu}^{\alpha}G^{\mu\nu}\psi_{-\nu}^{\beta} ,$$
(19)

where the fields $\psi^{\alpha}_{\pm\mu}$ are defined as $\psi^{\alpha}_{\pm\mu} = \psi^{\alpha}_{\mu} \pm \bar{\psi}^{\alpha}_{\mu}$. The superdeterminant s det M_{AB} is proportional to det $G^{eff}_{\mu\nu}$, which is assumed to be different from zero. Consequently, all constraints are of the second class and we can solve them explicitly.

III. TYPE I SUPERSTRING AS EFFECTIVE THEORY

The solution of the constraint equations $\Gamma_{\mu} = 0$, $\Gamma^{\alpha} = 0$ and $\Gamma^{\pi}_{\alpha} = 0$ has the form

$$x^{\mu}(\sigma) = q^{\mu} - 2\Theta^{\mu\nu} \int_0^{\sigma} d\sigma_1 p_{\nu} + \frac{\Theta^{\mu\alpha}}{2} \int_0^{\sigma} d\sigma_1 (p_{\alpha} + \bar{p}_{\alpha}),$$

$$\theta^{\alpha}(\sigma) = \eta^{\alpha} - \Theta^{\mu\alpha} \int_{0}^{\sigma} d\sigma_{1} p_{\mu} - \frac{\Theta^{\alpha\beta}}{4} \int_{0}^{\sigma} d\sigma_{1} (p_{\beta} + \bar{p}_{\beta}),$$

$$\bar{\theta}^{\alpha}(\sigma) = \bar{\eta}^{\alpha} - \Theta^{\mu\alpha} \int_{0}^{\sigma} d\sigma_{1} p_{\mu} - \frac{\Theta^{\alpha\beta}}{4} \int_{0}^{\sigma} d\sigma_{1} (p_{\beta} + \bar{p}_{\beta}) ,$$

$$\pi_{\mu} = p_{\mu}, \qquad \pi_{\alpha} = \frac{p_{\alpha}}{2}, \qquad \bar{\pi}_{\alpha} = \frac{\bar{p}_{\alpha}}{2}, \qquad (20)$$

where

$$\eta^{\alpha} \equiv \frac{1}{2} (\theta^{\alpha} + \Omega \bar{\theta}^{\alpha}) \,, \quad \bar{\eta}^{\alpha} \equiv \frac{1}{2} (\Omega \theta^{\alpha} + \bar{\theta}^{\alpha}) \,,$$

$$p_{\alpha} \equiv \pi_{\alpha} + \Omega \bar{\pi}_{\alpha} \,, \quad \bar{p}_{\alpha} \equiv \Omega \pi_{\alpha} + \bar{\pi}_{\alpha} \,, \tag{21}$$

and

$$\begin{split} \Theta^{\mu\nu} &= -\frac{1}{\kappa} (G_{eff}^{-1} B G^{-1})^{\mu\nu} \,, \\ \Theta^{\mu\alpha} &= 2 \Theta^{\mu\nu} (\Psi_{eff})^{\alpha}_{\nu} - \frac{1}{2\kappa} G^{\mu\nu} \psi^{\alpha}_{-\nu} \,, \\ \Theta^{\alpha\beta} &= \frac{1}{2\kappa} F_{s}^{\alpha\beta} + 4 (\Psi_{eff})^{\alpha}_{\mu} \Theta^{\mu\nu} (\Psi_{eff})^{\beta}_{\nu} \\ &- \frac{1}{\kappa} \psi^{\alpha}_{-\mu} (G^{-1} B G^{-1})^{\mu\nu} \psi^{\beta}_{-\nu} \\ &+ \frac{G^{\mu\nu}}{\kappa} \left[\psi^{\alpha}_{-\mu} (\Psi_{eff})^{\beta}_{\nu} + \psi^{\beta}_{-\mu} (\Psi_{eff})^{\alpha}_{\nu} \right] \,. \end{split}$$
 (22)

Substituting the solutions of the constraints (20) into the expression for canonical Hamiltonian (4) we obtain effective one. It easily produces the effective Lagrangian, which on the equations of motion for momentum p_{μ} , turns into

$$\mathcal{L}^{eff} = \frac{\kappa}{2} G^{eff}_{\mu\nu} \eta^{ab} \partial_a q^{\mu} \partial_b q^{\nu} +$$

$$- \pi_{\alpha} (\partial_{\tau} - \partial_{\sigma}) \left[\eta^{\alpha} + (\Psi_{eff})^{\alpha}_{\mu} q^{\mu} \right]$$

$$+ (\partial_{\tau} + \partial_{\sigma}) \left[\bar{\eta}^{\alpha} + (\Psi_{eff})^{\alpha}_{\mu} q^{\mu} \right] \bar{\pi}_{\alpha} + \frac{1}{2\kappa} \pi_{\alpha} F^{\alpha\beta}_{eff} \bar{\pi}_{\beta} .$$

We are going to find under what conditions the initial Lagrangian (3) produces the effective one. We can achieve that if we replace initial variables x^{μ} , θ^{α} and $\bar{\theta}^{\alpha}$ with corresponding effective ones q^{μ} , η^{α} and $\bar{\eta}^{\alpha}$, (momenta independent parts of their solution (20)) and make substitution

$$G_{\mu\nu} \rightarrow G_{\mu\nu}^{eff}, \quad \psi_{+\mu}^{\alpha} \rightarrow 2(\Psi_{eff})_{\mu}^{\alpha}, \quad F_{a}^{\alpha\beta} \rightarrow F_{eff}^{\alpha\beta},$$

 $B_{\mu\nu} \rightarrow 0, \qquad \psi_{-\mu}^{\alpha} \rightarrow 0, \qquad F_{s}^{\alpha\beta} \rightarrow 0, \quad (23)$

where $G_{\mu\nu}^{eff}$, $(\Psi_{eff})_{\mu}^{\alpha}$ and $F_{eff}^{\alpha\beta}$ are defined in (19). Note that $F_{eff}^{\alpha\beta}$ is antisymmetric by definition.

Consequently, in effective theory only Ω symmetric parts survive: graviton $G_{\mu\nu}$ in the NS-NS sector, gravitino $\psi^{\alpha}_{+\mu}$ in NS-R sector and antisymmetric part of R-R field strength, $F^{\alpha\beta}_a$. In our approach the effective theory has been obtained from initial IIB one on the solution of boundary conditions. As its Ω symmetric part, it corresponds to the type I superstring theory, but with improved background fields. As a consequence of boundary conditions at $\sigma = \pi$, this is a closed string theory.

IV. CONCLUDING REMARKS

In this paper we considered Green-Schwarz formulation of the type IIB superstring introduced in Ref.[3]. Using canonical method, following [5], we derived boundary conditions from the requirement that Hamiltonian, as time translation generator, has well defined functional derivatives in supercoordinates and canonically conjugated supermomenta. For bosonic coordinates x^{μ} we

chose Neumann boundary conditions, while for same chirality fermionic coordinates θ^{α} and $\bar{\theta}^{\alpha}$, in accordance with [3], we chose that $(\theta^{\alpha} - \bar{\theta}^{\alpha})|_{0}^{\pi} = 0$. As a consequence, the corresponding fermionic canonical momenta are equal at the boundary, $(\pi_{\alpha} - \bar{\pi}_{\alpha})|_{0}^{\pi} = 0$.

All boundary conditions at string endpoints we treated as canonical constraints. After Dirac consistency procedure, the infinite sets of the constraints at the endpoints can be rewritten in σ -dependent form. The constraints at $\sigma = \pi$ and $\sigma = 0$ are equal if we require 2π periodicity of all canonical variables. So, the theory obtained on the solution of boundary conditions is closed string theory.

Solving the constraints at $\sigma=0$ we obtained the expressions for supercoordinates x^{μ} , θ^{α} and $\bar{\theta}^{\alpha}$ in terms of effective ones q^{μ} , η^{α} and $\bar{\eta}^{\alpha}$ and their canonically conjugated momenta. The theory obtained from initial IIB superstring theory on the solution of constraints we will call effective theory. Effective Lagrangian is symmetric under orientfold projection Ω and consequently, it corresponds to the type I closed superstring theory. It propagates in effective background obtained from the type IIB one by substitution (23) and (19), or in terms of tensors instead of R-R field strength by substitution (2). The first terms in effective backgrounds are just standard Ω projections (unoriented part of IIB superstring theory) and second parts are our improvement.

Consequently, the type I superstring can not recognize explicitly oriented parts of IIB theory: NS-NS antisymmetric field $B_{\mu\nu}$, difference between two gravitinos $\psi^{\alpha}_{-\mu}$ and R-R fields: scalar C_0 and four rank antisymmetric tensor $C_{\mu\nu\rho\sigma}$, but it can see $B_{\mu\nu}$ and $\psi^{\alpha}_{-\mu}$ implicitly through effective backgrounds $G^{eff}_{\mu\nu}$, $(\Psi_{eff})^{\alpha}_{\mu}$ and $F^{eff}_{\mu\nu\rho}$.

We can formulate our result in other way that we can express the closed type I superstring theory in the form of open type IIB superstring theory, with appropriate choice of boundary conditions (9)-(10) and appropriate relation between background fields (19). Note that type IIB background fields are not uniquely defined by type I ones. For fixed type I theory, there is a class of corresponding IIB theories defined by different background $G_{\mu\nu}$, $B_{\mu\nu}$, $\psi^{\alpha}_{-\mu}$ and $F^{\alpha\beta}_a$ which produce the same $G^{eff}_{\mu\nu}$, $(\Psi_{eff})^{\alpha}_{\mu}$ and $F^{\alpha\beta}_{eff}$. Let us shortly discuss noncommutative properties of

Let us shortly discuss noncommutative properties of the open type IIB superstring. On the solutions of the boundary conditions (20) original string variables depend both on effective coordinates and effective momenta. This is a source of noncommutativity relations

$$\begin{aligned}
&\{x^{\mu}(\sigma), x^{\nu}(\bar{\sigma})\} &= 2\Theta^{\mu\nu}\theta(\sigma + \bar{\sigma}), \\
&\{x^{\mu}(\sigma), \theta^{\alpha}(\bar{\sigma})\} &= -\Theta^{\mu\alpha}\theta(\sigma + \bar{\sigma}), \\
&\{\theta^{\alpha}(\sigma), \bar{\theta}^{\beta}(\bar{\sigma})\} &= \frac{1}{2}\Theta^{\alpha\beta}\theta(\sigma + \bar{\sigma}),
\end{aligned} (24)$$

where $\theta(\sigma + \bar{\sigma})$ is the step function. In the case when the same chirality gravitinos ψ^{α}_{μ} and $\bar{\psi}^{\alpha}_{\mu}$ are equal, it follows that $\psi^{\alpha}_{-\mu} = 0$ and $(\Psi_{eff})^{\alpha}_{\mu} = \psi^{\alpha}_{\mu}$, so that we reproduct the results of Ref.[3]. More details about noncommutativity of type IIB superstring we will publish elsewhere.

Therefore, the background fields odd under Ω transformation have two roles. They are source of noncommutativity of supercoordinates and two of them, $B_{\mu\nu}$ and $\psi^{\alpha}_{-\mu}$, contribute to type I superstring background as bilinear combinations.

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